האם בכל זאת ניתן להכליל חקר מקרה?

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המחלקה לחינוך למדע וטכנולוגיה

הטכניון – מכון טכנולוגי לישראל

19.11.2014

“Case study and generalisation”

“In this paper we will begin by examining the arguments for naturalistic generalization and transferability [of case studies’ results]…We will suggest that to deny the possibility of case studies providing the basis for empirical generalizations is to accept the views of their critics too readily. We also point out that, in practice, much case study research has in fact put forward empirical generalizations… We will illustrate these points with examples from case study research in education, but we claim that our arguments are applicable more widely!” (p. 98)

Hammersley, M.; Foster, P. and Gomm, R. (2000). Case study and generalisation. In: Gomm, R.; Hammersley, M. and Foster, P. eds. *Case Study Method: Key Issues, Key Texts.* London: Sage, pp. 98–115.

The strength of case studies

1. They can help us understand complex inter-relationships
2. Case studies are grounded in “lived reality”
3. Case studies facilitate the exploration of the unexpected and unusual
4. Multiple case studies can enable research to focus on the significance of the idiosyncratic
5. Case studies can show the processes involved in causal relationships
6. Case studies can facilitate rich conceptual/theoretical development

Hodkinson & Hodkinson (2001)

The limitations of case studies

1. There is too much data for easy analysis
2. Very expensive, if attempted on a large scale
3. The complexity examined is difficult to represent simply
4. They do not lend themselves to numerical representation
5. They are not generalisable in the conventional sense
6. They are strongest when researcher's expertise and intuition are maximised, but this raises doubts about their "objectivity"
7. They are easy to dismiss, by those who do not like the messages that they contain
8. They cannot answer a large number of relevant and appropriate research questions

Hodkinson & Hodkinson (2001)

Anyway, how can case study findings be “generalised” - 1

1. Theory can be transposed beyond the original sites of study

“Where case studies generate new thinking, that thinking has a validity that does not entirely depend upon the cases from which it is drawn. Our theory of career decision making can be judged in other contexts, and other settings. It has already been used on two other studies [Ref.]. It can also be compared with and judged against other rival theories of career development and career decision making, elsewhere in the literature. It is much, much more than ‘the story of 10 trainees’”.

Hodkinson & Hodkinson (2001)

Anyway, how can case study findings be "generalised" - 2

2. Findings can 'ring true' in other settings

"Readers of case study research can judge whether or not the analysis presented sounds convincing, based upon what they know of similar situations and circumstances. In our experience, many professionals can empathise with our studies, because they recognise the sorts of situation that we describe. They seem less relevant/attractive to policy makers and some managers, perhaps because they reveal some of the over-simplifications upon which some policies and managerial practices are based."

Hodkinson & Hodkinson (2001)

Anyway, how can case study findings be “generalised” - 3

3. Case studies can provide provisional truths

“We would argue that our career decision-making theory establishes a provisional truth, even though it was based on the experiences of 10 young people, all white and all on a youth training programme. It is arguably the best account of such decision making in the current literature, and should stand, until contradictory findings or better theorising has been developed. The latter is beginning to happen, for example in the Ball et. al. (2000) study.”

Hodkinson & Hodkinson (2001)

Case study of …what/who?

Title	Based on	Citations
Problem Solving as a Basis for Reform in Curriculum and Instruction: The case of mathematics (Hiebert et al., 1996)	Theoretical/historical analysis	512
The effectiveness of "teach for America" and other under-certified teachers on student academic achievement: A case of harmful public policy (Laczko-Kerr & Berliner, 2002)	Statistical analysis of a sample of 293 teachers	372
The Development of the Idea of Mathematical Proof: A 5-Year Case Study (Maher and Martino, 1996)	A single student, 11 episodes during 5 years	192
Practice Meets Theory in Technology Education: A case of authentic learning in the high school setting (Hill & Smith, 1998)	A single teacher, one one-semester course	55
Reflections and Deflections of Policy: The case of Carol Turner (Ball, 1990)	A single teacher, one class	180

Examples from Google Scholar, Nov 18, 2014

Example 1

Maher, C. A., & Martino, A. M. (1996). The development of the idea of mathematical proof: A 5-year case study. *Journal for Research in Mathematics Education*, 27(2), 194-214. Cited by 192.

A single student, 11 episodes during 5 years

From Conclusions:

“In studying a young child's development of the idea of mathematical proof, we have gained some insight into the processes by which children may learn to make proofs” …

From Implications:

” The longitudinal case study of Stephanie is one step in documenting how a student was successfully empowered in building the idea of mathematical proof over time. Other studies in progress with larger groups of students suggest similar patterns. Replication of these studies will be important in advancing our understanding of young children's development of the idea of proof”(Maher and Martino, 1996, p. 212)

Example 2

Hill, A. M. & Smith, H. (1998). Practice Meets Theory in Technology Education: A Case of Authentic Learning in the High School Setting, *E-Journal of Technology Education* 9(2). Cited by 55.

A single teacher, one one-semester course

From Introduction:

"As is true of all research methodologies, case studies have both strengths and weaknesses. One perceived weakness, the inability to generalize findings to other settings, was not a concern given the prime objective of the present research: to specify how multiple factors function together in a particular dynamic situation to produce an apparently successful program in technology education."…"

From Discussion:

"Without claiming to generalize the results of this case study to other settings, we were able to generate a number of questions and implications for educational practice and theory…Other school subjects might apply this model to see if student attitudes and learning are enhanced in their content areas."

Example 3

Ball, D. L. (1990). Reflections and deflections of policy: The case of Carol Turner. *Educational Evaluation and Policy Analysis*, *12*, 247–259. Cited by 180.

A single teacher, one class, episodes during a school year

From the abstract:

"Would a state like California be happy if it could move all teachers to where Carol is? Alternatively, do the state policymakers want to change all teachers-those in the mainstream and on the fringe? The case highlights the complexity of the changes implied by California's Curriculum Framework and the difficulties inherent in communicating those changes in ways that can influence both pedestrian and accomplished practice." (Ball, 1990, p. 247).

Example 4

Koichu, B. (2012). Enhancing an intellectual need for defining and proving: A case of impossible objects. *For the Learning of Mathematics*, 32(1), 2-7.

A single teacher, a group of 8 students, two lessons

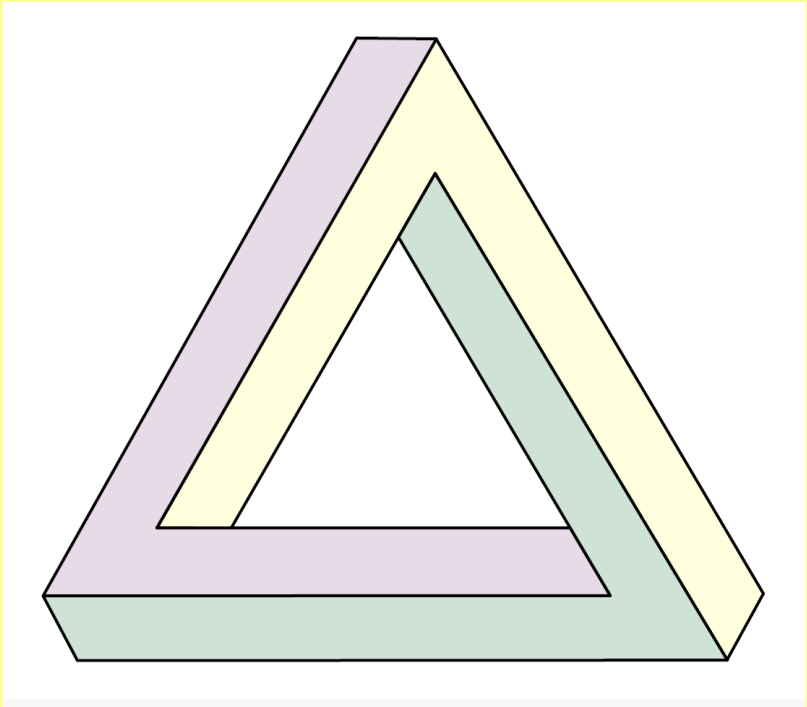
The last sentence of the paper:

"In closing, it is clear that the case presented in this paper does not allow to make generalizations. Nevertheless, I have formulated the above list of mathematical and didactical conditions in a somewhat decontextualized way. This is because I believe that the identified configuration of conditions can be adapted to different contexts, including regular school classrooms, and hope that future research will empirically substantiate them." (p. 7)

A story behind the last phrase

Is it impossible indeed?

An **impossible object** is a type of optical illusion consisting of a two-dimensional figure which is instantly and subconsciously interpreted by the visual system as representing a projection of a three-dimensional object although it is not actually possible for such an object to exist (at least not in the form interpreted by the visual system) (Wikipedia).

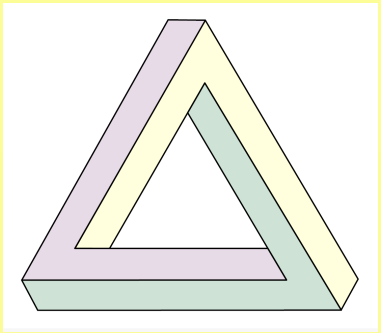


Penrose triangle

"It looks impossible, but it's possible. Cool!"

"I still don't get how the top corner may look as if it was attached…Great illusion!"

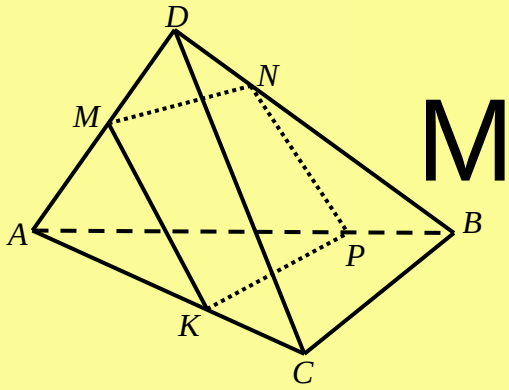
"The 'impossible' figure can really be made, no illusions, just real, they are possible to make!"



retrieved from the Mighty Optical Illusions Website

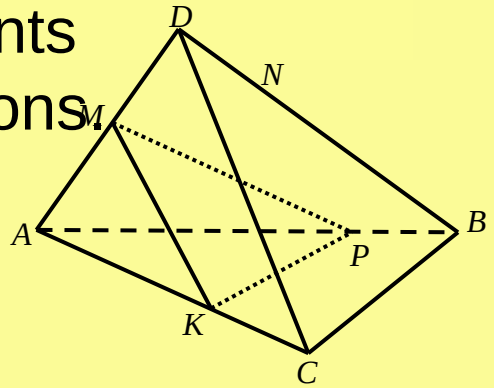
The case of interest

- **Content**: Exploration of the impossibility of the Penrose tribar with a group of 8 pre-service mathematics teachers at the course “Selected problems in mathematics” at the Technion.
- **Data**: from two meetings (4 hours) devoted to the exploration
- **Research goal**: To identify mathematical and didactical conditions for enabling the preservice teachers to autonomously construct a *proof-generated definition* (i.e., a definition that steams from proving properties of the explored objects).
- **Data analysis**: the focus on the students’ actions, in connection to the instructor’s pedagogical choices and interventions, with particular attention to the question whether the interventions preserve or suppress the students’ autonomous learning.



Mathematical outcomes

- Two valid proofs of the impossibility of the Penrose tribar were offered by the participants. The proofs relied on the definition of the Penrose tribar suggested by the students for both communicational and proof-oriented reasons (“Without this condition my proof wouldn't work”)
- The students “discovered” an axiom of the intersection of two planes, which was needed in order to accomplish the proof.
- The activity was developing so that the students autonomously reached the intended conclusions.



Findings: Configuration of conditions

(1) uncertainty regarding the material existence of the explored (ostensive) object opened a window for introducing theoretical considerations;

(2) the situation of uncertainty was set up so that the students started proving in order to remove their own real doubts;

(3) it was important that the participants had already possessed a deductive proof scheme;

(4) defining and proving interlaced and epistemologically supported each other during the activity.

(5) when the students could not make progress autonomously, they were helped by means of an intervention in the form of an auxiliary problem.

The last phrase before revision:

"In closing, it is obvious that the presented in this paper case does not allow us making generalizations. Still, we tried to formulate the above list of mathematical and didactical conditions in a somewhat decontextualized way. This is because we believe that the identified configuration of conditions can be adapted to additional situations and contexts and hope that future research will empirically substantiate this belief."

From the anonymous reviewer:

Having a rather small group of pre-service teachers reflecting about Penrose tribar is one thing — and the author properly acknowledge that the presented case "does not allow making generalizations". But then, what about a standard classroom? Something must be discussed and said about the possible adaptation to the ordinary classroom, maybe just as a discussion in the conclusion, as an incentive about what could come after, regarding the research project.

The last phrase after revision:

In closing, it is clear that the case presented in this paper does not allow to make generalizations. Nevertheless, I have formulated the above list of mathematical and didactical conditions in a somewhat decontextualized way. This is because I believe that the identified configuration of conditions can be adapted to different contexts, including regular school classrooms, and hope that future research will empirically substantiate them." (p. 7)

Follow up

The lesson of the same design was conducted at the same course in 2012.

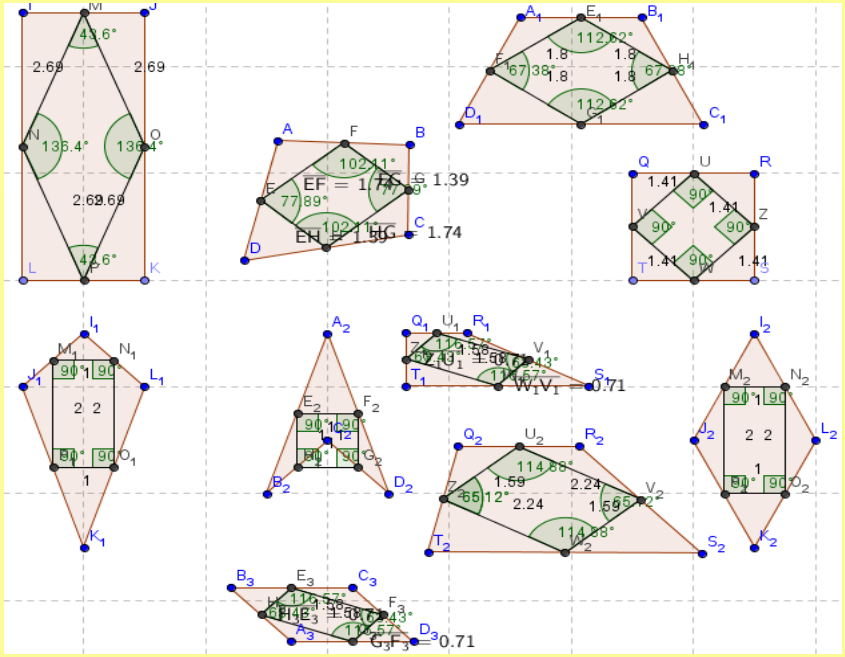
It led to very different results.

But some of the conditions were different.

Example 5

Lachmy, R., & Koichu, B. (2014). The interplay of empirical and deductive reasoning in proving "if" and "only if" statements in a Dynamic Geometry environment. *Journal of Mathematical Behavior*. Available online from August 22, 2014. doi:10.1016/j.jmathb.2014.07.002

Single student, a mathematical activity in on-line discussion forum



The interplay of empirical and deductive reasoning in proving "if" and "only if“ statements in a Dynamic Geometry environment

The research goal before revision:

In order to characterize the interplay of deductive and empirical arguments in a middle school student's reasoning while she worked with her classmates on formulating and justifying "if" and "only if" statements in a DG-supported online learning environment (we shall call the focus student Betty), we address the following research questions

1.Which claims did Betty formulate in response to the tasks aimed at the discovery of "if" and "only if" statements in the context of Varignon's theorem?

2.What were the processes (ascending, descending or probably others) Betty went through while formulating and justifying her claims? In particular, what were the types of warrants that Betty used in order to support her claims?

3.What were the hierarchies of dependencies and the types of invariants of Betty's DG drawings? How were the hierarchies of dependencies and types of invariants reflected in her empirical and deductive arguments in relation to the "if" and "only if" claims?

From one of the reviews:

- Reviewer 1: I appreciate that the authors exercised modesty in formulating research questions with respect to the student who was the focus of the case study, which is consistent with the case study methodology. At the same time, the goal of the case study is to shed light on more global and general phenomena that extrapolate beyond the case. I would appreciate the authors addressing these issues in general, in the form of an overarching research question or a statement of the research goals, before formulating the specific research questions (with respect to Betty). Otherwise there's a feeling that the whole study was conducted solely in order to learn something about Betty.

Research goal in the final version:

The goal of our study was to characterize the interplay of deductive and empirical arguments produced by mathematics learners in DG-supported learning environments, with special attention to mathematical structure of the statements to be discovered and justified. In line with many past studies having compatible goals (see concrete examples in Section 3.2), we find a case study methodology appropriate for achieving this goal. In this paper we chose to focus on a single student, Betty, whose posts represented well the mathematical ideas raised in three parallel groups working on the same tasks. All the names are pseudonyms.

לסיכום (ביניים):
האם בכל זאת ניתן להכליל חקר מקרה?

אם מאוד רוצים – כן. הכיצד?

- בשיטת ההתנצלות (שיטת "אבל...") (Hill & Smith)
- בשיטה "אסור לחוקר אך מותר לקורא ": רומזים לקוראים שהם יכולים להכליל בעצמם (Maher & Martino)
- בשיטת ההשוואה: מציעים לקוראים להשוות את המקרה המוצג עם מקרים אחרים (Hodkinson & Hodkinson, Ball, Lachmy & Koichu)
- בשיטה "השלכות תיאורטיות" (Hodkinson & Hodkinson, Maher & Martino, Koichu)
- בשיטה "תאמינו בכלליות התוצאות עד לרגע שמישהו ימצא תוצאות סותרות את התוצאות שלי" (Hodkinson & Hodkinson)

לסיכום, מכלילים , אבל בעקיפין

Virtual duoethnography: Another approach to generalization

Based on:

Zazkis, R., & Koichu, B. Virtual duoethnography: A simulated dialogue on infinitude of primes. *Educational Studies in Mathematics (in press)*.

Technion Mathematics Education Group
January 14, 2014

I am going…

to present a story of emergence a novel qualitative research approach and ask:

Do you see how this approach can be used/useful/powerful in your own research?

- If yes – how?
- If not – why not?

Theorem: There exist infinitely many prime numbers

Euclid's Proof.

Suppose that $p_1 = 2 < p_2 = 3 < \ldots < p_r$ are all the primes.

Let $P = p_1 p_2 \ldots p_r + 1$ and let p be a prime dividing P; then p cannot be any of $p_1, p_2, \ldots p_r$. Otherwise p would divide the difference $P - p_1, p_2, \ldots p_r = 1$. Which is impossible. So this prime p is still another prime, and $p_1, p_2, \ldots p_r$ would not be all the primes.

(from Ribenboim, 1996, p.3)

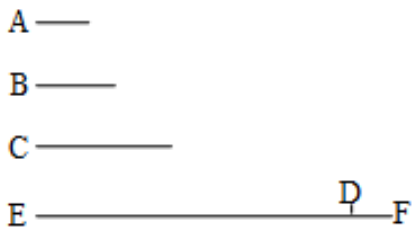
Euclid, Proposition 20-IX

Prime numbers are more than any assigned multitude of prime numbers.

Let *A*, *B*, and *C* be the assigned prime numbers.

I say that there are more prime numbers than *A*, *B*, and *C*.

For let the least number measured by *A*, *B*, and *C* be taken, and let it be DE; let the unit *DF* be added to *DE*.



Then *EF* is either prime or not.

First, let it be prime; then the prime numbers *A*, *B*, *C*, *EF* have been found which are more than *A*, *B*, and *C*.

Next, let *EF* not be prime; therefore it is measured by some prime number:

Let it be measured by the prime number *G*.

I say that *G* is not the same with any of the numbers *A*, *B*, and *C*.

For, If possible, let it be so.

Now *A*, *B*, and *C* measure *DE*; therefore *G* also will measure *DE*. But it also measures *EF*. Therefore *G*, being a number, will measure the remainder, the unit *DF*: which is absurd.

Therefore *G* is not the same with any one of the numbers *A*, *B*, and *C*.

And by hypothesis it is prime.

Therefore the prime numbers *A*, *B*, *C*, *G* have been found which are more than the assigned multitude of *A*, *B*, and *C*.

Q.E.D.

The task

Imagine two persons who love mathematics though they are not professional mathematicians, Euclidio and Goldboom. Euclidio lived in the time of Euclid (i.e., about 300 BC) and Goldboom – in the time of Goldbach (i.e., in the first half of the 18th century). They meet at heaven and discuss their favorite mathematical theorem, Proposition 20 Book IX of Euclid's *Elements*.

Euclidio understands perfectly the proof of the theorem given by Euclid, but for Goldboom it constitutes a difficulty. What would be the issues of difficulty for Goldboom in Euclid's proof? How could Euclidio help Goldboom, taking into consideration the difference between their mathematical backgrounds?

In your submission:

(a) Make Euclidio and Goldboom living characters as a good writer would do. That is, invent some details of their biographies (i.e., who they are? what is their relation to mathematics? why are they interested in primes? etc.), and, most importantly, describe their possible mathematical backgrounds and experiences: What might they know? Which books could influence them? Which mathematicians might they meet?

(b) Write a paragraph on what you believe could be a "problematic point" (or several points) in the Euclid's proof for Goldboom.

(c) Write an imaginary dialogue between Euclidio (E) and Goldboom (G), in which they discuss the Euclid's proof and Euclidio acts as a teacher (THIS IS THE MAIN PART OF THE TASK).

(d) Add a commentary to several lines in the dialogue that you created, explaining your choices of questions and answers.

Data: Students' submissions

Example 1: On numbers and units

G: The most troubling issue is that the theorem talks about numbers, but the proof discusses line segments and their measures. I simply do not understand the connection…

E: But those exactly are the numbers!

G: I have a feeling that you look at numbers in a very different way from what I used to. Can you explain, how numbers are defined for you?

E: Let's see. You first take a "unit", that is, a segment with a unit of lengths that serves as a basis for constructing other segments. A number is simply a segment that contains in it this "unit" several times, one next to another.

G: I believe now I understand. Whatever I call three and denote as 3 it is simply a line segment with the length of three times the unit.

Our research questions

When interpreting the given historical proof in the form of a dialogue between two fictional characters of solid but historically different mathematical backgrounds,

1. What do script writers identify as presumed points of difficulty that require an explanation?
2. What explanations are constructed by the participants in order to address the perceived difficulties?

Data analysis

- Dissecting the student-written submissions into excerpts, each of which began with a request to explain something and ended with a manifestation of understanding;
- Identifying, for each excerpt, an objects of understanding and ways of understanding;
- Categorizing and distilling widespread phenomena;
- Cross-checking the analysis and the categories;
- Answering the research questions and making conclusions.

Koichu, B., & Zazkis, R. Dialogical presentation of a historical proof:
Focus on “I understand” talk. Submitted to *Journal of Mathematical Behavior*.

Findings

Three types of potential difficulty attended to by the research participants in the context of the given historical proof.

Ways of understanding of particular mathematical concepts

Strategies of making sense a historical text

Affordances and limitations of the task

Our problem with “standard” approach

The “standard” format of data presentation was too narrow for us in order to discuss the participants’ ways of understanding in light of our own ways of understanding.

In simple words, we felt that “standard” approach did not give us enough room to present interesting things that we saw in the proofs, but did not show up in the data.

Thus, we engaged in the task of writing a dialogue by ourselves.

From the dialogue written by Rina and Boris

4.2.2 On number and unit

G: Let us start from the start. The proposition talks about numbers, but I do not see any numbers written or denoted.

E: When we are unsure about definition, we just look it up. One newcomer said that anything you can look up on Google. But I turn to *Elements*. *A number is a multitude composed of units*, this is by Definition 2 Book VII.

G: Units? Help me make sense if these. Unit, I think, is number 1, right?

E: *A unit is that by virtue of which each of the things that exist is called one*. This is Definition 1 Book VII. But why have you said "number 1"?

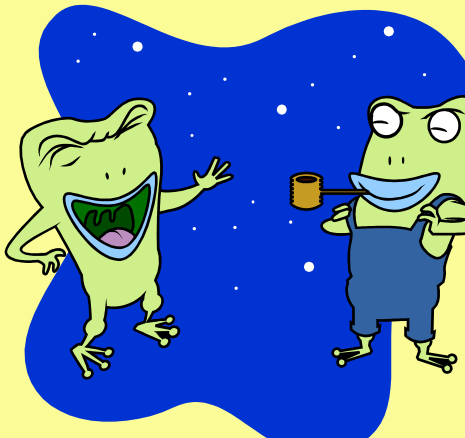
G: That is how I made sense of your unit, it represents the number 1. So the segments that you drew are composed of units and that is how you view numbers. Fine. But now you used a new term, what is 'by virtue'?

E: Unit is called one, but by definition it is not a number, we need a multitude of units to make a number. And virtue, my friend, does not require a definition, as only by virtue you found yourself in Heaven.

Virtual duoethnography (VDE)

VDE: a research approach, in which researchers produce a text of a dialogic format in the voices of fictional characters. The dialogue reflects the researchers' experiences gained when exploring a particular topic or phenomenon from different perspectives. These experiences can be integrated with outcomes of a particular empirical study or with any primary or secondary data sources.

According to Mus (2012), “unconstrained by a dataset, fiction has the means to articulate a radically subjective perspective without committing epistemic violence” (p.147).



Theoretical underpinnings-1

Duoethnography (Norris & Sawyer, 2012): a qualitative research methodology, in which researchers collaboratively produce a text of a dialogic format, in their own voices. The text juxtaposes their recollections of personal histories related to a phenomenon of importance and presents multiple views on this phenomenon.

- The goal of the methodology is to assist readers in recalling and reconceptualizing their own stories.
- “Duoethnographers are the sites of the research, not the topics.” (N&S, 2012). That is, duoethnography seeks to identify, describe, characterize and explain phenomena of importance, as many other types of qualitative research do.
- A duoethnography text is of a dialogical format, but it is rarely a script of any actual dialogue. The final text can be a result of a purposeful juxtaposing and intertwining stories that could have initially been told separately and then structured.
- A duoethnography text not only presents multiple views on a phenomenon, but presents them in development. In this way, the researchers put themselves in a vulnerable position, but simultaneously get room for exposing their personal research journeys and learning

Theoretical underpinnings-2

Virtual monologue (Leron & Hazzan, 1997): a tool, in which an experienced person (e.g., a researcher or a teacher) produces a text of a monologic format, in student's voice given in the first person language. The text is an expression of the person's view, based on his or her own learning, teaching or research experience, on what might be going on in the student's mind during a problem solving situation.

- Virtual monologue is not claimed to be a valid description of students' thinking, but a useful tool for communication and reflection.
- It can be created without data.
- It has been used as a tool in mathematics teacher professional development (Ejersbo & Leron, 2005; Ejersbo & Leron, 2010).

Theoretical underpinnings-3

Lakatos' dialogue: the dialogue, interspersed by comments and footnotes, between student-characters (Alfa, Beta, Gamma, Delta and so on) and a teacher-character (Teacher). The characters explore the question related to Descartes-Euler conjecture "F-E+V=2“

- The dialogue reads as a vivid conversation, but has an explicit structure: it is divided into parts entitled to denote which mathematical or meta-mathematical topics they are devoted to.
- It is based on historical data.
- It is not advocated as an instructional approach, but as a way that makes explicit the reasons underlying the introduced definitions and proof (Pimm, Beisiegel & Meglis, 2008)

VDE vs. other approaches

- VDE vs. duoethnography: in VDE, not necessarily two researchers are involved; it is not necessarily based on the researchers' personal stories (fictional dialogues)
- VDE vs. virtual monologue: VDE can be rooted in "real" data; it can be a dialogue, not only a monologue
- VDE vs. Lakatos' dialogue: Lakatos' dialogue is an example of VDE, which is rooted in historical data. VDE can use other types of data (or be used on personal stories and recollections as well).

Essential elements of VDE

- The choice of the characters and the attribution of particular statements to particular characters allow the researchers to present multiple perspectives on the chosen topic and elaborate on various subtle issues.
- A virtual duoethnography text gives room for presentation of what has showed up in "real data" as well as of what has not showed up, but might have showed up in some "ideal" case. The reader of the text can easily distinguish between these two types of statements either from the text itself or from the researcher-written comments on the text. Thus, a virtual duoethnography research approach provides a platform for expressing personal mathematical elaborations and pedagogical preferences without overshadowing the data (cf. Mus, 2012).
- The text presents students' errors as work in progress and the researchers' early (and sometimes naive) expectations and interpretations as stages of personal growth.

VDE and generalization

Mason (1998, 2002, 2010):

- The main enterprise of mathematics education researchers is to learn about themselves through interactions with others.
- The researchers can do so, in particular, by presenting to others *brief-and-vivid recollections* of their observations in ways that would resonate or trigger the others' own recollections.

VDE is generalizable to the extent the reader of VDE finds that it is generalizable.

Our “big hope”

VDE will be accepted by mathematics education research community as a legitimate qualitative research approach.

- That is, VDE will be accepted as a format for communicating research findings.
- We hope that VDE can be used in studies, in which the primary data is obtained from interviews, observations in a classroom, etc.).

Do you see how VDE approach can be used/useful/powerful in your own research?

- If yes – how?
- If not – why not?